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Correlator of the quark scalar currents and $\Gamma_{tot}(H \to$ hadrons) at $\mathcal{O}(\alpha_s^3)$ in pQCD

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Abstract

We present the results of the analytical evaluation of the massless four-loop $\mathcal{O}(\alpha_s^3)$ correction to the correlator of the quark scalar currents and the Higgs decay rate into hadrons. In numerical form we found (in $\overline{\mathrm{MS}}$ scheme) that $\Gamma(H \to \overline{b}b) = \frac{3G_F}{4\sqrt{2}\pi} M_H m_b^2(M_H) \left[1 + 5.667\alpha_s/\pi + (35.94 - 1.359 n_f)(\alpha_s/\pi)^2 + \left(164.139 - 25.771 n_f + 0.259 n_f^2\right)(\alpha_s/\pi)^3\right]$ where n_f is the number of quark favours and $\alpha_s = \alpha_s(M_H)$.

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1 Introduction

In the Standard Model (SM) one physical scalar Higgs boson is present as a remnant of the mechanism of mass generation. Particularly interesting for the observation of the Higgs boson with an intermediate mass $M_H < 2M_W$ is the dominant decay channel into a bottom pair $H \to b\bar{b}$. (Standard Model properties of the Higgs boson have been discussed in many reviews; see for example [1, 2]). The partial width $\Gamma(H \to b\bar{b})$ is significantly affected by QCD radiative corrections. First order α_s corrections including the full m_b dependence were studied by several groups [3, 4, 5, 6, 7]. Second order corrections were calculated in the limit $m_b^2 \ll M_H^2$. Apart from the trivial overall factor m_b^2 due to the Yukawa coupling, corrections were obtained for otherwise massless quarks in Refs. [8] and for a nonvanishing mass of the virtual top quark in Ref. [9] (both results are confirmed in Ref. [10]). Subleading corrections in the m_b^2/M_H^2 expansion were found in Ref. [11] and also confirmed in Ref. [10]. In the latter work an additional quasi-massless (and numerically important) contributions of order α_s^2 have been identified and elaborated. They come from so-called singlet diagrams with a non-decoupling top quark loop inside (similar effects for the decay of the Z-boson to quarks were first discovered in [12]). The results of Ref. [10] have been confirmed and extended by computing extra terms in the expansion in M_H^2/m_t^2 in Ref. [13].

In this work we compute the next-next-to-leading massless correction of order α_s^3 to $\Gamma(H \to \text{hadrons})$.

2 Preliminaries

We start with considering the two-point correlators

$$\Pi^{\rm S}_{ff'}(Q^2) = (4\pi)^2 i \int dx e^{iqx} \langle 0|\, T[\, J^{\rm S}_f(x) J^{\rm S}_{f'}(0)\,]\, |0\rangle. \tag{1}$$

Here $Q^2 = -q^2$, $J^{\rm S}_f = \bar{\Psi}_f \Psi_f$ is the scalar current for quarks with flavour f and mass m_f, which are coupled to the scalar Higgs bosons. The total hadronic decay rate of a scalar Higgs boson is naturally expressed in terms of the absorptive part of the correlator (1)

$$R^{\rm S}_{ff'}(s) = \frac{1}{2\pi s} \mathrm{Im}\, \Pi^{\rm S}_{ff'}(-s - i\epsilon),$$

as follows [3, 4, 5]:

$$\Gamma(H \to \text{hadrons}) = \frac{G_F}{4\sqrt{2}\pi} M_H \sum_{ff'} m_f m_{f'} R^{\rm S}_{ff'}(s = M_H^2). \tag{2}$$

Assuming that the Higgs boson mass M_H is less then $2m_t$ and much larger than m_b, we shall treat correlators (1) within the massless five-quark QCD. (In fact, even in the limit of heavy top the Higgs boson coupling with the current $J^{\rm S}_t$ still contributes to Γ_H; we shall ignore these effects in our work. For a recent discussion see e.g. Ref. [14].) Under such an assumption the nondiagonal correlators $\Pi^{\rm S}_{ff'}$ with $f \neq f'$ vanish identically while the diagonal correlators get flavour independent, that is

$$\Pi^{\rm S}_{ff}(Q^2) \equiv \Pi^{\rm S}(Q^2) \quad \text{and} \qquad R^{\rm S}_{ff}(s) \equiv R^{\rm S}(s).$$

The renormalization mode of the polarization operator $\Pi^{\rm S}(Q^2)$ reads (see, e.g. Ref. [15])

$$\Pi^{\rm S}(Q^2, a_s, \mu^2) = Z_q^{\rm SS} Q^2 + Z_m^2 \Pi_0^{\rm S}(Q^2, a_s^0)], \tag{3}$$

where $a_s = \alpha_s/\pi = g^2/(4\pi^2)$, g is the strong coupling constant; Z_m is the quark mass renormalization constant. Within the $\overline{\rm MS}$ scheme [16]

$$Z_q^{\rm SS} = \sum_{1 \le j \le i} \left(Z_q^{\rm SS} \right)_{ij} \frac{a_s^{i-1}}{\epsilon^j} \tag{4}$$

and the coefficients $\left(Z_q^{\rm SS} \right)_{ij}$ are just numbers, with $D = 4 - 2\epsilon$ standing for the space-time dimension. As a result we arrive at the following renormalization group (RG) equation for the polarization operator $\Pi^{\rm S}(Q^2)$

$$\left(\mu^2 \frac{\partial}{\partial \mu^2} + a_s \beta(a_s) \frac{\partial}{\partial a_s} + 2\gamma_m(a_s) \right) \Pi^{\rm S} = \gamma_q^{\rm SS}(a_s) \tag{5}$$

or, equivalently, ($L_Q = \ln \frac{\mu^2}{Q^2}$)

$$\frac{\partial}{\partial L_Q} \Pi^{\rm S} = \gamma_q^{\rm SS}(a_s) - \left(2\gamma_m(a_s) + \beta(a_s) a_s \frac{\partial}{\partial a_s} \right) \Pi^{\rm S}. \tag{6}$$

Here the anomalous dimensions $\gamma_q^{\rm SS}(a_s)$ and $\gamma_m(a_s)$, and the β-function $\beta(a_s)$ are defined in the usual way

$$\gamma_q^{\rm SS} = \mu^2 \frac{d}{d\mu^2} (Z_q^{\rm SS}) - \epsilon Z_q^{\rm SS} = - \sum_{i \ge 0} (i+1) (Z_q^{\rm SS})_{i1} a_s^i, \tag{7}$$

$$\mu^2 \frac{d}{d\mu^2} a_s = \alpha_s \beta(a_s) \equiv - \sum_{i \ge 0} \beta_i a_s^{i+2}, \quad \mu^2 \frac{d}{d\mu^2} \bar{m}(\mu) = \bar{m}(\mu) \gamma_m(a_s) \equiv -\bar{m} \sum_{i \ge 0} \gamma_m^i \left(a_s \right)^{i+1}. \tag{8}$$

The relation (6) demonstrates explicitly the main computational advantage of finding at first the polarization function $\Pi^{\rm S}(Q^2)$ against a direct calculation of $R^{\rm S}(s)$ in the case of massless pQCD. Indeed, in order a_s^3 the derivative $\frac{\partial}{\partial L_Q} \Pi^{\rm S}$ and, consequently, $R^{\rm S}(s)$ depends on the very function $\Pi^{\rm S}$ which is multiplied by at least one factor of a_s. Thus, one needs only to know $\Pi^{\rm S}$ to order a_s^2 and the anomalous dimension $\gamma_q^{\rm SS}(a_s)$ to order a_s^3 to find all Q-dependent terms in $\Pi^{\rm S}$ at $\mathcal{O}(a_s^3)$, since the beta function and the quark mass anomalous dimension γ_m are reliably known to order a_s^3 from Refs. [17-20].

As we shall see later both problems are eventually reduceable to calculation of a specific class of diagrams representing some massless Feynman integrals (FI) depending on only one external momentum (to be named *p-integrals* below) and with the number of loops not exceeding three. Note that this is in an obvious disagreement with a statement from Ref. [11] about the inevitable necessity to compute *finite* parts of *four-loop* p-integrals in order to find the $\mathcal{O}(a_s^3)$ contribution to $R^{\rm S}(s)$. Yet, we agree with the author of Ref. [11] that if such a point were correct it would certainly preclude, at least at the present state of the art, any possibility of analytical calculation of $R^{\rm S}(s)$ to order α_s^3.

3 Calculation of $\Pi^{\rm S}$

In order α_s^2 the polarization operator $\Pi^{\rm S}$ is contributed by twelve three-loop p-integrals. Such a calculation is now to be considered as almost trivial one because of three facts:

a There is an elaborated algorithm which provides a way to evaluate analytically divergent as well as finite parts of any three-loop dimensionally regulated p-integral [21, 22].

b The algorithm is reliably implemented in the language of FORM [23] as the package named MINCER in Ref. [24].

c The package has been extensively tested and a chance of a bug in it seems to be very small.

We have computed the $\Pi_0^{\rm S}$ to α_s^2 using the general gauge with the gluon propagator $(g_{\mu\nu} - \xi\frac{q_\mu q_\nu}{q^2})/q^2$. On performing the renormalization according to eq. (3) we have obtained the following result:

$$
\begin{aligned}
\Pi^{\rm SS}(Q^2) \;=\;& d[R]Q^2\left\{-4 - 2\ln\frac{\mu^2}{Q^2} + a_s\,C_F\left[-\frac{131}{8} + 6\,\zeta(3) - \frac{17}{2}\ln\frac{\mu^2}{Q^2} - \frac{3}{2}\ln^2\frac{\mu^2}{Q^2}\right]\right.\\
&+a_s^2\Bigg[\quad C_F^2\left(-\frac{1613}{64} + 24\,\zeta(3) - \frac{9}{4}\zeta(4) - 15\,\zeta(5) - \frac{691}{32}\ln\frac{\mu^2}{Q^2}\right.\\
&\qquad\left.+\frac{9}{2}\zeta(3)\ln\frac{\mu^2}{Q^2} - \frac{105}{16}\ln^2\frac{\mu^2}{Q^2} - \frac{3}{4}\ln^3\frac{\mu^2}{Q^2}\right)\\
&\quad + C_F\,C_A\left(-\frac{14419}{288} + \frac{75}{4}\zeta(3) + \frac{9}{8}\zeta(4) + \frac{5}{2}\zeta(5) - \frac{893}{32}\ln\frac{\mu^2}{Q^2}\right.\\
&\qquad\left.+\frac{31}{4}\zeta(3)\ln\frac{\mu^2}{Q^2} - \frac{71}{12}\ln^2\frac{\mu^2}{Q^2} - \frac{11}{24}\ln^3\frac{\mu^2}{Q^2}\right)\\
&\quad \left.+ C_F\,T\,n_f\left(\frac{511}{36} - 4\,\zeta(3) + \frac{65}{8}\ln\frac{\mu^2}{Q^2} - 2\,\zeta(3)\ln\frac{\mu^2}{Q^2} + \frac{11}{6}\ln^2\frac{\mu^2}{Q^2} + \frac{1}{6}\ln^3\frac{\mu^2}{Q^2}\right)\Bigg]\right\}.
\end{aligned}
\tag{9}
$$

Here $a_s = \alpha_s(\mu)/\pi$, C_A and C_F are the Casimir operators of the adjoint and quark (defining) representations of the colour group; T is the normalization of the trace of generators of quark representation $Tr(t^a t^b) = T\delta^{ab}$; n_f is the number of quark flavours; $d[R]$ is the dimension of the quark representation of the colour group. Note, that all the Q-dependent terms in (9) are in agreement with the results of [8]. The independence of (9) from the gauge parameter ξ is of course expected on general grounds.

4 Calculation of the anomalous dimension $\gamma_q^{\rm SS}$

There is about a hundred of diagrams contributing to $\Pi^{\rm S}$ in order α_s^3. According to (3), to compute $\gamma_q^{\rm SS}$ one should determine the UV counterterms (that is essentially divergent parts) of all the corresponding four-loop p-integrals. Unfortunately, at present there exists no way to compute directly the divergent part of a generic four-loop p-integral.

The only available (unfortunately rather involved) approach to perform such calculations analytically is to use the method of Infrared Rearrangement (IRR) discovered by A. Vladimirov in Ref. [25] (see also Refs. [26]). The method simplifies calculation of UV counterterms by the effective use of the following important theorem: in a class of minimal renormalization schemes (including $\overline{\mathrm{MS}}$- and MS-ones) any UV counterterm has to be polynomial in external momenta *and* masses [27]. It amounts to an appropriate transformation of the IR structure of FI's by expanding the latter in a formal Taylor series with respect to some external momenta and masses, with resulting FI's being much simpler to calculate.

The method of IRR was significantly extended with elaborating a so-called R^*-operation in Refs. [28, 29, 30]. By an explicit construction of the corresponding algorithm, it has been shown in Ref. [29] how to reduce calculation of the UV counterterm for an arbitrary (h+1)-loop FI to evaluation of divergent and finite parts of some appropriately chosen h-loop p-integrals.

It should be stressed that the R^*-operation is absolutely essential for the algorithm to work in general case, though in most (but not in all) practical cases one could proceed without it. However, such a practice forces the use of a diagram-wise renormalization procedure; the latter, being very difficult to perform by a computer, implies a huge amount of highly error-prone and time-consuming manipulations with hundreds of diagrams.

For instance, the only QCD four-loop problem performed by now is the evaluation of the ratio $R(s) = \sigma_{\mathrm{tot}}(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ to order α_s^3. It was done in Refs. [31, 32] within the diagram-wise renormalization approach. The core of the problem is the calculation of the four-loop contribution to the photon anomalous dimension entering into the RG equation for the photon polarization operator. The initial 98 four-loop diagrams contributing to the photon polarization operator proliferate to about 250 after the IRR procedure is applied. In addition these diagrams contain about 600 various subdiagrams which should be computed separately in order to subtract UV subdivergences.

Technically, the calculation of γ_q^{SS} is obviously very similar to that of the photon anomalous dimension. Consequently, any attempt of straightforward repetition of those calculations for the case of the scalar correlator would mean a few man-years of routine and boring work and, thus, would be not acceptable for the present author.

Below we will use, instead, the power of the R^*-operation and simplify the application of IRR to the problem so far that both UV and IR renormalizations can be done in a global form and, consequently, can be simply performed by computer.

We begin with from the Dayson–Schwinger equation for the correlator (1) written in the bare form[1]

$$\Pi_0^{\mathrm{S}}(q, a_s^0) = -\int \mathrm{d}p \frac{(4\pi)^2}{(2\pi)^D} Tr[G^0(p+q, a_s^0)\Gamma_S^0(p, q, a_s^0)G^0(p, a_s^0)]. \tag{10}$$

Here G^0 and Γ_S^0 are the full quark propagator and the scalar current vertex function respectively; below the integration with respect to the loop momentum p with the weight function $\frac{(4\pi)^2}{(2\pi)^D}$ will be only understood but not explicitly displayed.

[1] For simplicity we set the $'$t Hooft-Veltman unit of mass μ equal to 1 below.

The renormalized version of (10) reads

$$\Pi^{\rm S}(q,a_s) = Z_q^{\rm SS} Q^2 - \frac{Z_S^2}{Z_2^2} Tr[G^0(p+q,a_s^0)\Gamma_S^0(p,q,a_s^0)G^0(p,a_s^0)]. \qquad (11)$$

Here Z_2 is the quark wave function renormalization constant; $Z_{\rm S}$ is the renormalization constant of the scalar quark current defined as

$$[\overline{\psi}\psi] = Z_{\rm S}/Z_2 \ \overline{\psi^0}\psi^0,$$

where the current inside squared brackets is the renormalized one. A well-known equality $Z_{\rm S} = Z_m Z_2$ implies the equivalence of (3) and (11).

From the finiteness of the renormalized correlator one gets

$$\begin{aligned} Z_q^{\rm SS} &= -K_\epsilon \left\{ \frac{1}{2D} \left(\frac{Z_{\rm S}}{Z_2^2} \Box_q Tr[G^0(p+q,a_s^0)\Gamma_S^0(p,q,a_s^0)\tilde{G}^0(p,a_s^0)] \right.\right. \\ &+ \left.\left. \frac{\delta Z_{\rm S} Z_{\rm S}}{Z_2^2} \Box_q Tr[G^0(p+q,a_s^0)\Gamma_S^0(p,q,a_s^0)G^0(p,a_s^0)] \right) \right\} \end{aligned} \qquad (12)$$

where $K_\epsilon f(\epsilon)$ stands for the singular part of the Laurent expansion of $f(\epsilon)$ in ϵ near $\epsilon = 0$ and $\delta Z_{\rm S} = Z_{\rm S} - 1$. In Eq. (12) we have let a Dalambertian with respect to the external momentum q act on quadratically divergent diagrams to transform them to the logarithmically divergent ones. We also have introduced an auxiliary mass dependence to a quark propagator — the one entering into the "left" current J_S— by making the following replacement

$$G^0(p,a_s^0) \to \tilde{G}^0(p,a_s^0)\ p^2/(p^2 - m_0^2). \qquad (13)$$

Note, please, that the auxiliary mass dependence has caused somewhat more complicated structure of UV renormalizations in the right hand side of Eq. (12).

The idea of the method of IRR is quite simple: since the renormalization constant $Z_q^{\rm SS}$ does not depend on anything dimensionful one could significantly simplify its calculation by nullifying the momentum q in Eq. (12). The only requirement which must be respected is the absence of any IR singularities in the resulting integrals. Unfortunately, a mass introduced to a propagator distinguished by some topological property like the one we created above is not always sufficient to suppress all IR divergences in all diagrams. For instance, if $q = 0$ then there appear completely massless tadpoles in the second term on the rhs of Eq. (12). Thus, the eq. (12) with $q = 0$ is *not valid* unless the unwanted IR poles are all identified and subtracted away.

This is certainly the job the R^*-operation was created for! The rules of Ref. [29] spell how to do it on the diagram-wise level. The only remaining problem is to disentangle the relevant combinatorics and put down the IR subtractions in a global form. The task is facilitated by the fact that, as shown in Ref .[30] the IR counterterms for an arbitrary diagram can be determined in terms of some properly chosen combination of the UV ones. The final formula incorporating all necessary UV and IR subtractions reads

$$\begin{aligned} Z_q^{\rm SS} &= -K_\epsilon \left\{ \frac{1}{2D} \frac{Z_{\rm S}}{Z_2^2} \Box_q Tr[G^0(p+q,a_s^0)\Gamma_S^0(p,q,a_s^0)\tilde{G}^0(p,a_s^0)]|_{q\,=\,0} \right. \\ &- \left. \frac{Z_{\rm S}}{Z_2^2}\frac{1}{4} Tr[\delta\Gamma_{\tilde{S}}^0(0,0,a_s^0)] \frac{Z_q^{\rm SS}}{Z_m^2} - \frac{\delta Z_{\rm S} Z_{\rm S} Z_q^{\rm SS}}{Z_2^2 Z_m^2} \right\}. \end{aligned} \qquad (14)$$

Here, by $\delta\Gamma^0_{\tilde S}(p,q,a^0_s)$ we denote the vertex function of the scalar current with the tree contribution removed. The "tilde" atop S again means that in every diagram the quark propagator entering to the vertex $J^{\rm S}$ is softened at small momenta by means of the auxiliary mass m_0 according to Eq. (13). The bare coupling constant a^0_s is to be understood as $a_s = Z_a a_s$, with Z_a being the coupling constant renormalization constant. To our accuracy in a_s

$$Z_a = 1 + a_s(-\beta_0/\epsilon) + a_s^2(\beta_0^2/\epsilon^2 - \beta_1/(2\epsilon))\,.$$

Finally, an inspection of (14) immediately shows that, in order to find the $(n+1)$-loop correction to $Z^{\rm SS}_q$, one needs only to know the renormalization constants $Z^{\rm SS}_q$, Z_2 and Z_m to order a^n_s as well as the bare Green functions

$$G^0(p,a^0_s),\quad \frac{\partial}{\partial q_\beta}[\Gamma^0_S(p,q,a^0_s)]\big|_{q\,=\,0},\quad \Box_q[\Gamma^0_S(p,q,a^0_s)]\big|_{q\,=\,0},\quad \delta\Gamma^0_{\tilde S}(0,0,a^0_s) \qquad (15)$$

up to (and including) n-loops. Thus, we have obtained a general formula for $Z^{\rm SS}_q$ in terms of bare p-integrals with explicitly resolved UV and IR subtractions.

5 Results and discussion

We have computed with the program MINCER [24] the unrenormalized three-loop Green functions (15) as well as the renormalization constants Z_m and Z_2 to order a^3_s. The calculations have been performed in the general covariant gauge. The total calculational time with an IBM workstation is about 40 hours for the general gauge; for the Feynman one it is reduced to about 4 hours.

Then we have used Eqs. (14) and (7) to find $\gamma^{\rm SS}_q$ to order a^3_s. Our results for $\gamma^{\rm SS}_q$ and $R^{\rm S}(s)$ read

$$\begin{aligned}
\gamma^{\rm SS}_q \quad &= \\
d[R]\;\Bigg\{ &-2 + a_s\,C_F\left[-\frac{5}{2}\right] + a_s^2\left[C_F^2\left(\frac{119}{32} - \frac{9}{2}\zeta(3)\right) + C_F\,C_A\left(-\frac{77}{16} + \frac{9}{4}\zeta(3)\right) + C_F\,T\,n_f\right] \\
+a_s^3\Bigg[\quad & C_F^3\left(-\frac{4651}{384} - \frac{29}{4}\zeta(3) + \frac{27}{8}\zeta(4) + \frac{45}{4}\zeta(5)\right) \\
&+ C_F^2\,C_A\left(\frac{641}{48} - \frac{259}{16}\zeta(3) + \frac{39}{16}\zeta(4) + \frac{45}{8}\zeta(5)\right) \\
&+ C_F C_A^2\left(-\frac{267889}{31104} + \frac{475}{48}\zeta(3) - \frac{33}{16}\zeta(4) - \frac{45}{8}\zeta(5)\right) \\
&+ C_F^2\,T\,n_f\left(\frac{125}{32} - \frac{1}{2}\zeta(3) - 3\,\zeta(4)\right) \\
&+ C_F\,C_A\,T\,n_f\left(\frac{631}{7776} + \frac{5}{3}\zeta(3) + \frac{9}{4}\zeta(4)\right) \\
&+ C_F T^2\,n_f^2\left(\frac{1625}{1944} - \frac{2}{3}\zeta(3)\right)\Bigg]\Bigg\}\,. \qquad (16)
\end{aligned}$$

and

$$R^{\mathrm{S}}(s,\mu) = d[R]\Big\{1 + a_s(\mu)\left[s_1 + 2\gamma_m^0 \ln\tfrac{\mu^2}{s}\right]
+ a_s^2(\mu)\left[s_2 + \ln\tfrac{\mu^2}{s}\left(s_1\beta_0 + 2s_1\gamma_m^0 + 2\gamma_m^1\right) + \ln^2\tfrac{\mu^2}{s}\left(\beta_0\gamma_m^0 + 2(\gamma_m^0)^2\right)\right]
+ a_s^3(\mu)\Big[s_3 + \ln\tfrac{\mu^2}{s}\left(2s_2\beta_0 + s_1\beta_1 + 2s_2\gamma_m^0 + 2s_1\gamma_m^1 + 2\gamma_m^2\right)
+ \ln^2\tfrac{\mu^2}{s}\left(s_1\beta_0^2 + 3s_1\beta_0\gamma_m^0 + \beta_1\gamma_m^0 + 2s_1(\gamma_m^0)^2 + 2\beta_0\gamma_m^1 + 4\gamma_m^0\gamma_m^1\right)
+ \ln^3\tfrac{\mu^2}{s}\left(\tfrac{2}{3}\beta_0^2\gamma_m^0 + 2\beta_0(\gamma_m^0)^2 + \tfrac{4}{3}(\gamma_m^0)^3\right)\Big]\Big\}. \quad (17)$$

Here the coefficients of the beta-function $\beta_0,\ \beta_1$ and the quark mass anomalous dimension $\gamma_m^i,\ \ i=0,1,2$ are defined according to (7,8) and read

$$\gamma_m^0 = \frac{1}{4}[3C_F], \qquad \gamma_m^1 = \frac{1}{16}\left[\frac{3}{2}C_F^2 + \frac{97}{6}C_FC_A - \frac{10}{3}C_FTn_f\right],$$

$$\gamma_m^2 = \frac{1}{64}\left[\frac{129}{2}C_F^3 - \frac{129}{4}C_F^2C_A + \frac{11413}{108}C_FC_A^2\right. \quad (18)$$

$$\left. + C_F^2Tn_f(48\zeta(3)-46) + C_FC_ATn_f\left(-48\zeta(3) - \frac{556}{27}\right) - \frac{140}{27}C_FT^2n_f^2\right],$$

$$\beta_0 = \frac{1}{4}\left[\frac{11}{3}C_A - \frac{4}{3}Tn_f\right], \qquad \beta_1 = \frac{1}{16}\left[\frac{34}{3}C_A^2 - 4C_FTn_f - \frac{20}{3}C_ATn_f\right]. \quad (19)$$

At last, the coefficients $s_1,\ s_2$ and s_3 are found to be

$$s_1 = C_F\left[\frac{17}{4}\right], \quad s_2 = C_F^2\left[\frac{691}{64} - \frac{3}{8}\pi^2 - \frac{9}{4}\zeta(3)\right] + C_F\,C_A\left[\frac{893}{64} - \frac{11}{48}\pi^2 - \frac{31}{8}\zeta(3)\right]$$
$$+ C_F\,T\,n_f\left[-\frac{65}{16} + \frac{1}{12}\pi^2 + \zeta(3)\right],$$

$$\begin{aligned}
s_3 = &\\
+ &\ C_F^3\left[\frac{23443}{768} - \frac{27}{16}\pi^2 - \frac{239}{16}\zeta(3) + \frac{45}{8}\zeta(5)\right]\\
+ &\ C_F^2\,C_A\left[\frac{13153}{192} - \frac{383}{96}\pi^2 - \frac{1089}{32}\zeta(3) + \frac{145}{16}\zeta(5)\right]\\
+ &\ C_FC_A^2\left[\frac{3894493}{62208} - \frac{1715}{864}\pi^2 - \frac{2329}{96}\zeta(3) + \frac{25}{48}\zeta(5)\right]\\
+ &\ C_F^2\,T\,n_f\left[-\frac{88}{3} + \frac{65}{48}\pi^2 + \frac{65}{4}\zeta(3) + \frac{3}{4}\zeta(4) - 5\,\zeta(5)\right]\\
+ &\ C_F\,C_A\,T\,n_f\left[-\frac{33475}{972} + \frac{571}{432}\pi^2 + \frac{22}{3}\zeta(3) - \frac{3}{4}\zeta(4) + \frac{5}{6}\zeta(5)\right]\\
+ &\ C_FT^2\,n_f^2\left[\frac{15511}{3888} - \frac{11}{54}\pi^2 - \zeta(3)\right].
\end{aligned} \quad (20)$$

We observe that neither γ_q^{SS} nor $R^{\mathrm{S}}(s)$ depend on the gauge fixing parameter ξ as it must be. For the standard QCD colour group values $d[R]=3$, $C_F=4/3$, $C_A=3$ and $T=1/2$

we get for $R^{\rm S}(s)$ with μ^2 set to s:

$$
\begin{aligned}
R^{\rm S}(s) \;=\; & 3\Big\{\, 1+a_s\left[\frac{17}{3}\right]+a_s^2\left[\frac{10801}{144}-\frac{19}{12}\pi^2-\frac{39}{2}\zeta(3)-\frac{65}{24}n_f+\frac{1}{18}\pi^2 n_f+\frac{2}{3}\zeta(3)\,n_f\right] \\
+a_s^3\Big[& \frac{6163613}{5184}-\frac{3535}{72}\pi^2-\frac{109735}{216}\zeta(3)+\frac{815}{12}\zeta(5)-\frac{46147}{486}n_f+\frac{277}{72}\pi^2 n_f \\
+ & \frac{262}{9}\zeta(3)\,n_f-\frac{5}{6}\zeta(4)\,n_f-\frac{25}{9}\zeta(5)\,n_f+\frac{15511}{11664}n_f^2-\frac{11}{162}\pi^2 n_f^2-\frac{1}{3}\zeta(3)\,n_f^2\Big]\Big\}\,,
\end{aligned}
\tag{21}
$$

or, in the numerical form,

$$
\begin{aligned}
R^{\rm S}(s)=3\,\Big\{ \quad 1 \;+\; & 5.66667\frac{\alpha_s(s)}{\pi}+(35.93996-1.35865n_f)\left(\frac{\alpha_s(s)}{\pi}\right)^2 \\
+\; & \left(164.13921-25.77119n_f+0.258974n_f^2\right)\left(\frac{\alpha_s(s)}{\pi}\right)^3\Big\}.
\end{aligned}
\tag{22}
$$

At last, for the phenomenologically relevant case of $n_f=5$ we obtain

$$
R^{\rm S}(s)=3\left\{1+5.66667\frac{\alpha_s(s)}{\pi}+29.1467\left(\frac{\alpha_s(s)}{\pi}\right)^2+41.7576\left(\frac{\alpha_s(s)}{\pi}\right)^3\right\}. \tag{23}
$$

Due to eq. (2) the combination $m_b^2 R^{\rm S}(s=M_H^2)$ is directly related to the Higgs decay rate to the $b\bar{b}$ pairs plus gluons. The corresponding expression including power suppressed corrections reads

$$
\begin{aligned}
\Gamma(H\to\bar{b}b) \;=\; & \\
\frac{3G_F}{4\sqrt{2}\pi}M_H m_b^2(M_H)\,\Big[\quad 1 \;+\; & 5.67\,a_s(M_H)+29.15\,a_s^2(M_H)+41.76\,a_s^3(M_H) \\
+\; & \frac{m_b^2(M_H)}{M_H^2}\left(-6-40\,a_s(M_H)-87.72\,a_s^2(M_H)\right)\Big].
\end{aligned}
\tag{24}
$$

Let us take as input parameters a $\Lambda^{(5)}_{QCD}=233$ MeV and a bottom pole mass of $m_b^{\rm pole}=4.7$ GeV. The latter translates into the running mass $m_b(M_H^2)=2.84/2.75/2.69$ GeV for Higgs masses of $M_H=70/100/130$ GeV. All other light quarks are assumed to be massless. One arrives at the following values for the strong coupling constant: $\alpha_s(M_H^2)=0.125/0.118/0.114$ corresponding to the three different values of M_H. The corresponding numerical values for different contributions to $R^{\rm S}(M_H^2)$ are given[2] in Table 1.
To summarize: we have suggested a new convenient way to compute the UV renormalization constant of the correlator of the scalar quark currents. Our final formula (14) directly

[2] It should be noted that this discussion is somewhat inconsistent because of the following reasons. First, the relation between the running and the pole masses is known only with the α_s^2 accuracy [33]. Second, one needs not yet available coefficients, γ_m^3 and β_3, for taking into account the terms of order $(\alpha_s(m_b^{pole})-\alpha_s(M_H))^3$ in the running of the quark mass and the coupling constant. Nevertheless, the numbers in Table 1 are correct for the given values of $\alpha_s(M_H)$ and $m_b(M_H)$.

M_H	$\alpha_s(M_H)$	$m_b(M_H^2)$	$\mathcal{O}\left(\alpha_s(M_H)\right)$	$\mathcal{O}\left(\alpha_s^2(M_H)\right)$	$\mathcal{O}\left(\alpha_s^3(M_H)\right)$
70 GeV	0.125	2.84 GeV	0.226 - 0.00262	0.0461 - 0.00023	0.0026
100 GeV	0.118	2.75 GeV	0.213 - 0.00114	0.0411 - 0.00009	0.0022
130 GeV	0.114	2.69 GeV	0.206 - 0.00062	0.0384 - 0.00005	0.0020

Table 1: The contributions of different orders to $R^{\mathrm{S}}/3$; the first number stands for the massless part and the second one for the power suppressed $\mathcal{O}(\frac{m_b^2}{M_H^2})$ part (where available).

expresses the constant in terms of unrenormalized p-integrals, with all UV and IR subtractions being implemented in a global form. The formula is ideally suited for carrying out completely automatic calculations and can be easily extended for the case of general bilinear quark currents. Detailed derivation of the formula will be presented elsewhere. Using the formula and the FORM version of the MINCER we have computed the $\mathcal{O}(\alpha_s^3)$ correction to the anomalous dimension γ_q^{SS}, to the absorptive part of the scalar correlator $R^{\mathrm{S}}(s)$ and to $\Gamma(H \to \text{hadrons})$ in pQCD. All the calculations have been performed with the use of the general covariant gauge. The gauge independence of the results constitutes a strong check of the correctness of our approach.

Numerically, the correction of order α_s^3 to $\Gamma(H \to \text{hadrons})$ proves to be relatively small. However, for the Higgs boson with intermediate mass it is more important than the power suppressed $\mathcal{O}(\frac{m_b^2}{M_H^2}\alpha_s^2)$ contribution. (see Table 1).

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